TIME-DEPENDENT ELECTRON TRANSPORT THROUGH THE MULTI-TERMINAL QUANTUM DOT

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ABSTRACT. We consider the time-dependent electron transport through a quantum dot connected to multiple leads in the presence of the additional over-dot (bridge) tunnelling channels by using the evolution operator technique. Each terminal and quantum dot are disturbed by an external oscillating field resulting in a time-dependence of the corresponding energy levels. The final analytical expressions for the currents flowing in the system are given assuming the wide-band limit approximation. We investigate also the transient-current characteristics in the case of the rectangular-pulse modulations imposed on the dot-lead barriers. The time-averaged current and its derivative with respect to the gate voltage have been calculated for a wide range of parameters.

1. Introduction

The advance of experimental techniques on a nanometer scale has enhanced the interest on the electronic transport through quantum dots. Especially interesting are the transport properties of a quantum dot (QD) under the influence of external time-dependent fields. The microwave fields applied to different parts of the system under consideration modify the QD charge and tunneling current. New effects have been observed and theoretically described, e.g. photon-assisted tunneling through small QD-s with well-resolved discrete energy states, photon-electron pumps and others [1]. One can investigate the current flowing through a QD under periodic (non-periodic) modulation of the tunneling barriers or under harmonic modulation of the electron energy levels in both (say, left and right) electron reservoirs, e.g. [2, 3]. One of the important problem of mesoscopic physics is the interference of the charge carriers. This interference appears when two (or more) transmission channels for tunneling electron exist. The experimental sit-
uation in which the destructive interference may occur can be realized in the scanning tunneling microscope (STM) or in multi-terminal QD system.

In this paper we generalize models existing in the literature and consider the QD connected with leads with additional over-dot (bridge) tunneling channel between leads, e.g. [4, 5, 6, 7]. The schematic picture of the system is shown in Fig. 1. The QD connected with three leads corresponds also to the possible STM experimental setup in which the QD placed between two leads - say the right leads, can be probed by means of the additional electrode (tip) - say the left lead. In such configuration the additional channels for the electron transfer between STM tip and the right and left leads exist. We consider the system driven out of the equilibrium by means of dc voltage bias and time-dependent external fields. To treat this nonequilibrium, time-dependent electron transport process we use the evolution operator technique and find the final expression for the current flowing in the system in terms of the appropriate matrix elements of this operator.

2. Model and calculation method

The Hamiltonian of the QD coupled through the tunneling barriers $V_{E_i,d}$ ($i = 1, 2, \ldots, N$) to $N$ metal leads with chemical potential $\mu_i$ can be written as $H = H_0(t) + V(t)$, where

$$H_0(t) = \sum_{i=1}^{N} \sum_{E_i} \varepsilon_{E_i}(t) c_{E_i}^+ c_{E_i} + \varepsilon_0(t) c_{d}^+ c_{d},$$

Fig. 1: Schematic picture of the multi-terminal QD system
\[ V(t) = \sum_{i,j=1}^{N} \sum_{\tilde{E}_i, \tilde{E}_j} V_{\tilde{E}_i, \tilde{E}_j}(t) c_{k_i}^\dagger c_{k_j} + \text{h.c.} + \sum_{i=1}^{N} \sum_{\tilde{E}_i} V_{\tilde{E}_i, d}(t) c_{k_i}^\dagger c_d + \text{h.c.} \]  

(2)

within usually used notation. For simplicity the dot is characterized only by the single level \( \varepsilon_0 \) and we have neglected the intra-dot electron-electron interaction.

We assume the microwave fields applied to the leads and the QD as follows:

\[ \varepsilon_{\tilde{E}_i}(t) = \varepsilon_{\tilde{E}_i} + \Delta_i \cos \omega t \]

\[ \varepsilon_0(t) = \varepsilon_0 + \Delta_0 \cos \omega t. \]

The dynamical evolution of the charge localized on the QD and the current flowing in the system can be described in terms of the time-evolution operator \( U(t, t_0) \), [5, 6, 7]. The QD charge is as follows:

\[ n_d(t) = n_d(t_0) |U_{dd}(t, t_0)|^2 + \sum_{i=1}^{N} \sum_{\tilde{E}_i} n_{\tilde{E}_i}(t_0) |U_{d\tilde{E}_i}(t, t_0)|^2, \]

(3)

and the tunneling current flowing from the \( i \)-th lead which can be obtained from the time derivative of the total number of electrons in this lead, \( j_i(t) = -edn_i(t)/dt \) (cf. [3]), where

\[ n_i(t) = \sum_{\tilde{E}_i} n_{\tilde{E}_i}(t) = \sum_{\tilde{E}_i} |n_d(t_0) U_{d\tilde{E}_i}(t, t_0)|^2 + \sum_{j=1}^{N} \sum_{\tilde{E}_j} n_{\tilde{E}_j}(t_0) |U_{\tilde{E}_j, \tilde{E}_i}(t, t_0)|^2. \]

(4)

Here \( U_{dd}(t, t_0) \), \( U_{d\tilde{E}_i}(t, t_0) \), \( U_{\tilde{E}_j, d}(t, t_0) \) and \( U_{\tilde{E}_j, \tilde{E}_i}(t, t_0) \) denote the matrix elements of \( U(t, t_0) \) calculated within the basis functions containing the single-particle functions \( |d\rangle \) and \( |\tilde{E}_i\rangle \) corresponding to the QD and \( i \)-th metal lead, respectively. \( n_d(t_0) \) and \( n_{\tilde{E}_i}(t_0) \) represent the initial filling of the corresponding single-particle states. The matrix elements of the evolution operator can be obtained using the equation of motion (in the interaction representation) [5, 6, 7]:

\[ i \frac{\partial}{\partial t} U(t, t_0) = \hat{V}(t) U(t, t_0), \]

(5)

where

\[ \hat{V}(t) = U_0(t, t_0) V(t) U_0^+(t, t_0), \]

(6)

and

\[ U_0(t, t_0) = T \exp \left( -i \int_{t_0}^{t} dt' H_0(t') \right). \]

(7)

According to Eq. (5) the matrix elements of the evolution operator satisfy the corresponding coupled set of the differential equations:

\[ \frac{\partial}{\partial t} U_{dd}(t, t_0) = -i \sum_{j=1}^{N} \sum_{\tilde{E}_j} \hat{V}_{d\tilde{E}_j}(t) U_{d\tilde{E}_j}(t, t_0), \]

(8)

\[ \frac{\partial}{\partial t} U_{\tilde{E}_j, d}(t, t_0) = -i \hat{V}_{\tilde{E}_j, d}(t) U_{dd}(t, t_0) - i \sum_{j=1}^{N} \sum_{\tilde{E}_j} \hat{V}_{\tilde{E}_j, \tilde{E}_j}(t) U_{d\tilde{E}_j}(t, t_0), \]

(9)
\[
\frac{\partial}{\partial t} U_{\vec{k}, \vec{k}'}(t, t_0) = -i \tilde{V}_{\vec{k}, \vec{k}'}(t) U_{\vec{d} \vec{k}'}(t, t_0) - i \sum_{\vec{k}_i} \tilde{V}_{\vec{k}, \vec{k}_i}(t) U_{\vec{d} \vec{k}_i}(t, t_0),
\]
(10)
\[
\frac{\partial}{\partial t} U_{\vec{d} \vec{k}'}(t, t_0) = -i \sum_{\vec{k}_i} \tilde{V}_{\vec{d} \vec{k}_i}(t) U_{\vec{e} \vec{k}_i}(t, t_0),
\]
(11)

where \( \vec{k}_i \) runs over the wave vectors \( \vec{k} \) characterizing the single-electron states of the \( i \)-th metal lead. The functions \( \tilde{V}_{\vec{d} \vec{k}'} \) and \( \tilde{V}_{\vec{e} \vec{k}_i} \), calculated according to Eqs (6-7) are as follows:
\[
\tilde{V}_{\vec{k}, \vec{k}'}(t) \equiv \langle \vec{k}' | \tilde{V}(t) | \vec{k} \rangle = V_{\vec{k}, \vec{k}'} \exp(i(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'})(t - t_0) - i(\Delta_{\vec{k}} - \Delta_{\vec{k}'})(\sin \omega t - \sin \omega t_0)/\omega,
\]
(12)
\[
\tilde{V}_{\vec{k}, \vec{k}_i}(t) \equiv \langle \vec{k}_i | \tilde{V}(t) | \vec{k} \rangle = V_{\vec{k}, \vec{k}_i} \exp(i(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}_i})(t - t_0) - i(\Delta_{\vec{k}} - \Delta_{\vec{k}_i})(\sin \omega t - \sin \omega t_0)/\omega.
\]
(13)

For the case of the QD coupled with two leads without the bridge between them, the solution of the corresponding set of equations is greatly simplified. For example, the integro-differential equations for \( U_{\vec{d} \vec{d}'}(t, t_0) \) and \( U_{\vec{d} \vec{e}'}(t, t_0) \) are as follows:
\[
\frac{\partial}{\partial t} U_{\vec{d} \vec{d}'}(t, t_0) = -\int_{t_0}^{t} dt_1 \mathcal{K}(t, t_1) U_{\vec{d} \vec{d}'}(t_1, t_0),
\]
(14)
\[
\frac{\partial}{\partial t} U_{\vec{d} \vec{e}'}(t, t_0) = -i \tilde{V}_{\vec{d} \vec{k}_i}(t) - \int_{t_0}^{t} dt_1 \mathcal{K}(t, t_1) U_{\vec{d} \vec{e}'}(t_1, t_0),
\]
(15)

where
\[
\mathcal{K}(t, t_1) = \sum_{i=1,2} \sum_{\vec{k}_i} |V_{\vec{d} \vec{k}_i}|^2 \mathcal{D}_i(t - t_1) \times \exp(i\varepsilon_{\vec{d}}(t - t_1) + i(\Delta_{\vec{d}} - \Delta_{\vec{d}_i})(\sin \omega t - \sin \omega t_1)/\omega).
\]
(16)

Here \( \mathcal{D}_i(t) \) is the Fourier transform of the \( i \)-th lead density of states. If we include the over-dot (bridge) coupling between leads the corresponding equations for the matrix elements of the evolution operator are more complicated, but assuming the widely used wide-band limit we obtain, for example for \( U_{\vec{d} \vec{d}'}(t, t_0) \):
\[
\frac{\partial}{\partial t} U_{\vec{d} \vec{d}'}(t, t_0) = -\Gamma \left( 1 + \sum_{j=1}^{\infty} (-i\pi V_{RL}/D)^j \right) U_{\vec{d} \vec{d}'}(t, t_0),
\]
(17)

with the solution
\[
U_{\vec{d} \vec{d}'}(t, t_0) = \exp(-C_1(t - t_0)).
\]
(18)

Here we assumed \( V_{\vec{k}, \vec{k}'} = V_{\vec{k}, \vec{k}'} \equiv V_{RL}, \Gamma = 2\pi V^2/D, \pi V_{RL}/D \equiv x < 1 \), \( C_1 = \Gamma/(1 + ix) \) and \( D \) denotes the width of the leads energy bands.
For the case of the QD coupled with three leads \((L, R_1, R_2)\) the corresponding equations for matrix elements of the evolution operator are more complicated but under the wide-band limit they are still tractable and for example, the current leaving the left lead takes the form:

\[
\langle j_L \rangle = \frac{2x^2}{(1 + 2x^2)^2} \left[ \frac{2\mu_L - \mu_{R_1} - \mu_{R_2}}{\Gamma} \right] \left[ \frac{1}{3} \left( 1 - 13x^2 + 4x^4 \right) \text{Im}\Phi + 2x(1 - 2x^2)\text{Re}\Phi \right],
\]

where

\[
\Phi = 2 \int f_L(\varepsilon)\langle A_L(\varepsilon) \rangle d\varepsilon - \sum_{i=R_1, R_2} \int f_i(\varepsilon)\langle A_i(\varepsilon) \rangle d\varepsilon,
\]

and

\[
\langle A_i(\varepsilon) \rangle = \sum_k J_k^2 \left( \frac{\Delta_0 - \Delta_k}{\omega} \right) \left( \varepsilon - \varepsilon_0 - \omega k + \frac{2\Gamma x}{1 + 2x^2} + i \frac{3\Gamma}{2(1 + 2x^2)} \right)^{-1}.
\]

Here \(J_k^2\) denotes the Bessel function. Note, that the formula for the averaged current, Eq. 19, is nearly exact - it has been obtained assuming only the wide-band limit approximation. In normal mesoscopic systems this approximation has been widely used as it is justifiable under the conditions which usually are met in experiments. Namely, the bandwidth of the leads should be much larger than \(\Gamma\) and the leads density of states and the hopping matrix elements \(V_{\tilde{g},i,\tilde{g}}\) and \(V_{\tilde{g},d}\) should vary slowly with the energy around the QD level.

3. Results and discussion

We consider the QD coupled with two and three metal leads with the additional over-dot (bridge) couplings between leads. The time-dependent currents are calculated in the case when the periodic rectangular-pulse external field is applied to each QD-lead barrier. In such a case we integrate numerically the corresponding set of the differential equations for the matrix elements of the evolution operator. We consider also the time-averaged values for currents and conductance in the case of harmonic modulation of the system parameters. We assume the temperature \(T = 0\) K and we take for \(V_{\tilde{g},i,\tilde{g}}\) the values comparable with \(V_{\tilde{g},d}\). We estimated \(V_{\tilde{g},d}\) (assuming its \(\tilde{g}\)-independence, \(V_{\tilde{g},d} \equiv V_{\tilde{g}} = V\)) using the relation \(\Gamma_\alpha = 2\pi |V_\alpha|^2/D_\alpha\), where \(D_\alpha\) is the \(\alpha\)-lead’s bandwidth and \(D_\alpha = 100\ \Gamma_\alpha\) \((\Gamma_L = \Gamma_R = \Gamma; D_L = D_R = D\) was assumed). In our calculations we assumed \(\varepsilon = 1\), all energies are given in \(\Gamma\) units, time in \(\hbar/\Gamma\) units, the current, its derivatives and frequency are given in \(e\Gamma/\hbar, c^2\Gamma/\hbar\) and \(\Gamma/\hbar\) units, respectively.

In Fig. 2 we show the currents for the QD coupled with three leads \(R_1, R_2\) and \(L\) with the additional couplings \(V_{LR_i}\) and \(V_{LR_2}\). The barriers QD-\(R_1\) lead and QD-\(R_2\) lead are changing in time according to a periodic rectangular-pulse.
Fig. 2: The time-dependent current flowing in the system of a QD coupled with three leads: L, R1 and R2. The L-lead is coupled with the QD only - the left panels and with the QD and two other leads, \( V_{LR_1} = V_{LR_2} = 4 \) - the right panels. The couplings between the QD and \( R_1, R_2 \) leads is changed periodically. The upper, middle and lower panels correspond to \( \mu_L = 3, 0 \) and \(-3\), respectively. \( \mu_{R_1} = -\mu_{R_2} = 3, \varepsilon_d = 0 \). The thin, thick and broken curves correspond to \( j_L, j_{R_1} \) and \( j_{R_2} \) currents, respectively.

external field with the period \( T = 10 \), which is applied to each barrier and these two fields are out of phase with a phase difference of \( \pi \). The third lead (L) is connected with the QD through the time-independent barrier. We checked that the QD charge hardly depends on the additional \( V_{LR} \) couplings. Although the QD charge is almost insensitive to the additional over-dot couplings the currents demonstrate such dependence. Especially visible are the differences for the case when the chemical potential \( \mu_L \) of the third electrode L lies between chemical potentials of two other leads, see Fig. 2 B, E. For other values of \( \mu_L \) (relative to \( \mu_{R_1} \) and \( \mu_{R_2} \)) the influence of the over-dot tunneling channels considered here is smaller. Note, that after the abrupt changing of the coupling strength the currents \( j_L, j_{R_1} \) and \( j_{R_2} \) are rapidly changed too, and after a short time reach the steady values. The QD coupled with three leads could be considered as the three-state system. As we see in Fig. 2 by changing the coupling strength the current changes its value from e.g. zero to the positive (negative) value or from the negative to the positive value and vice versa, depending on the chemical potentials of all leads. The additional couplings between leads modify the currents flowing in this
Fig. 3: The time-averaged current $\langle j_L \rangle$ flowing from the left lead (upper panels) and its derivative with respect to $\varepsilon_0$ (lower panels) as a function of $\varepsilon_0$ and $\mu_L$. The left panels correspond to the QD coupled with two leads: $\Delta_L = 8$, $\Delta_R = 4$, $\Delta_R = 2$, $\mu_R = 0$ and the right panels correspond to three-terminal system: $\Delta_L = 8$, $\Delta_R = 4$, $\Delta_R = 2$, $\Delta_R = -2$, $\mu_R = 0$, $\mu_R = -4$ and $\omega = 5$, $V_{LR} = V_{LR} = 0$, $\Gamma = 1$.

In the next step we show results for time-averaged current and its derivative with respect to $\varepsilon_0$, calculated as the functions of $\varepsilon_0$ and $\mu_L$, for the case of the QD coupled with two leads, Fig. 3A,C and coupled with three leads, Fig. 3B,D. In principle, inclusion of the third electrode does not introduce significant changes to the current $\langle j_L \rangle$, and only the dependence on the gate voltage is more "diffusive". More transparent changes are visible on $d\langle j_L \rangle/d\varepsilon_0$ curves, calculated as functions of $\varepsilon_0$ and $\mu_L$ (Fig. 3C,D). Now, for the three-terminal QD system we observe three distinct enhancements regions going along $\varepsilon_0$-axis at constant $\mu_L$ (see, for example, the changes at $\mu_L = 10$) whereas for the QD coupled with two electrodes we observe only one corresponding peak.

More dramatic differences can be observed in the three-terminal QD system when we introduce changes for parameters of the one electrode, only. In Fig. 4 we show $\langle j_L \rangle$ and $d\langle j_L \rangle/d\varepsilon_0$ for two different sets of parameters. The panels A and C correspond to the system in which one of the right lead is not affected by the time-dependent field, $\Delta_R = 0$, whereas the panels B and D show results for the case $\Delta_R = 2$. The results presented on the panels A and C are, in fact, very
Fig. 4: The time-averaged current $\langle j_L \rangle$ (upper panels) and its derivative with respect to $\varepsilon_0$ (lower panels) as a function of $\varepsilon_0$ and the frequency $\omega$. $\Delta_L = 8$, $\Delta_0 = 4$, $\Delta_{R_1} = 0$, $\mu_L = 0.2$, $\mu_{R_1} = -0.2$, and $\mu_{R_2} = 0$. The amplitudes of the oscillation of the third electrode are $\Delta_{R_2} = 0$ (left panels) and $\Delta_{R_2} = 2$ (right panels). $V_{L,R_1} = V_{L,R_2} = 0$, $\mu = 1$.

similar to the results obtained for two-terminal QD system with $\Delta_L = 8$, $\Delta_0 = 4$ and $\Delta_{R_1} = 0$ (not shown here). So, depending on the amplitude of oscillations of the microwave field applied to the additional third electrode, the resulting current flowing out of the left electrode can differ in magnitude and functional dependence on $\varepsilon_0$ and $\omega$ from that ones for slightly different parameters of the third lead.

The simplest verification of the results obtained e.g. for the three-terminal QD system can be performed for the case when the microwave radiation is applied to the QD only. In such case, the current flowing in the two-terminal QD system, for small differences $\mu_L - \mu_R$, shows the satellite peaks (photon-assisted tunneling), localized at $\varepsilon_0 = \frac{\mu_L + \mu_R}{2} \pm n\omega$ (for vanishing over-dot coupling between leads) [1]. The amplitude and the half-width of the main and satellite peaks take the values $2\Gamma$ and $\frac{1}{2\pi} J_n^2 \left( \frac{\Delta}{\omega} \right) (\mu_L - \mu_R)$, respectively. For the three-terminal QD system the current flowing out from the L-lead (for small values of $\mu_L - \mu_R$, and $\mu_L - \mu_{R_1}$, and for $\mu_L > \mu_{R_1} > \mu_{R_2}$) shows approximately the same structure consisting of the main and satellite peaks of the half-width $3\Gamma$ and the amplitude $\frac{1}{2\pi} J_n^2 \left( \frac{\Delta}{\omega} \right) (2\mu_L - \mu_{R_1} - \mu_{R_2})$. Similarly, the conductance $d\langle j_L \rangle/d\mu_L$ shows the Lorentzian peaks of
the height $\frac{1}{2} \sqrt{\frac{\hbar}{\pi}} J_n^2 \left( \frac{\Delta_0}{\omega} \right)$ and half-width $2\Gamma$ in the case of the two-terminal QD system and of height $\frac{1}{2} \sqrt{\frac{\hbar}{\pi}} J_n^2 \left( \frac{\Delta_0}{\omega} \right)$ and the half-width $3\Gamma$ in the three-terminal QD system.

In summary, we considered the currents flowing in the system of the multi-terminal QD system using the evolution operator approach. The influence of the external harmonic microwave fields and rectangular-pulse modulations of the dot-lead barriers on the currents was investigated.

References